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Magnetohydrodynamic lubrication flow between parallel plates

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The magnetohydrodynamic lubrication flow in an externally pressurized thrust bearing is investigated both theoretically and experimentally. The ordinary magnetohydrodynamic lubrication theory for this bearing is extended to include fluid inertia effects. Very good agreement is obtained between theory and experiment.

1. Introduction

In recent years, there has been a considerable interest in magnetohydrodynamic lubrication theory. A number of analytical studies have been published, but the authors are aware of only one publication which deals with the experimental verification of these studies (Kuzma, Maki & Donnelly 1964).

The object of this paper is to present both the theory and experimental results for the magnetohydrodynamic lubrication flow between parallel plates, an interesting example of radial Hartmann flow. An analytical investigation of this flow has been carried out by Hughes & Elco (1962) in the lubrication approximation. The conditions of the experiment are not the same as those assumed by Hughes & Elco, making it necessary to extend their work to include the effects of fluid inertia. The results of the experiment are in good agreement with the extended theory.

2. Apparatus

A simplified sketch of the apparatus is shown in figure 1. Mercury enters the fluid inlet and fills a recess cut in the bottom plate. From there it flows radially outward between two plates under the influence of a vertical magnetic field. The top plate is made of glass and is 12 in. in diameter and 1·25 in. thick. The bottom plate is made of stainless steel with a 4 in. diameter recess cut in the plate. The diameter of the plate was selected so that the velocity profile would be fully developed over a large portion of the plates. A 2 in. diameter post in the centre of the recess supports the top plate. Interchangeable shims are used to separate the plates and maintain constant film thickness. The distribution of pressure in the flow is determined by a series of pressure taps spaced at 0·25 in. radial intervals on the lower plate and connected directly to mercury manometers. A photograph of the bottom plate is shown in figure 2 (plate 1). One can see the inlet to the central recess, the radial pressure taps, the collector trough and three mercury outlets.

A diagram of the fluid supply system is shown in figure 3. The pump is a screw impeller type powered by a two-horsepower motor. The heat generated by the pump is removed by a four pass tube-and-shell-type stainless steel heat

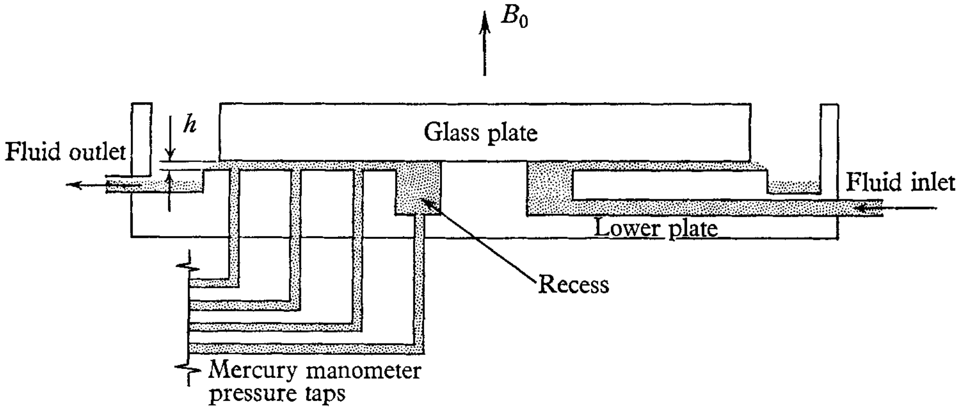

FIGURE 1. Simplified drawing of plates.

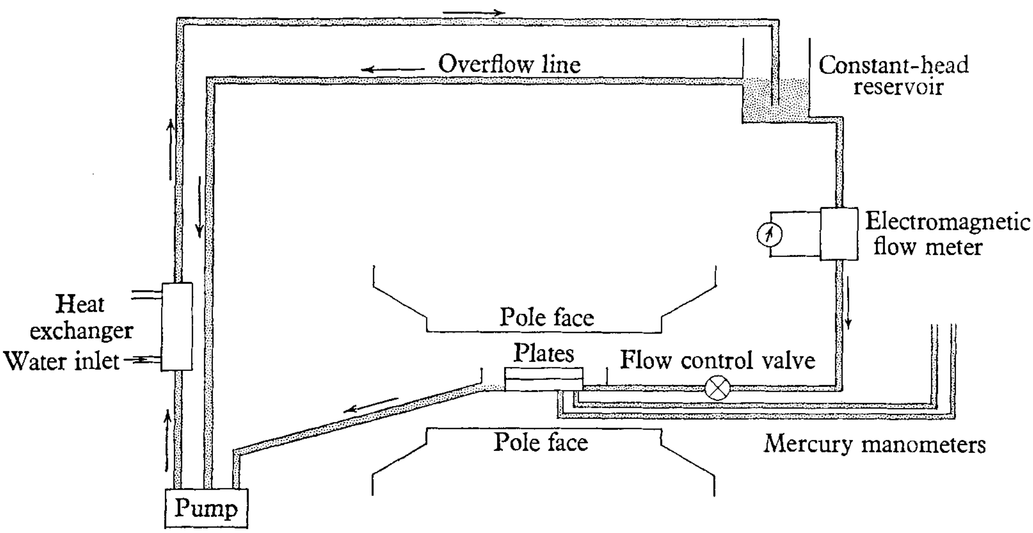

FIGURE 3. Diagram of mercury supply system.

exchanger. Mercury is pumped through the heat exchanger to a reservoir which is located approximately 48 in. above the level of the plates. A constant head is maintained in the reservoir by allowing the mercury to overflow back to the pump. From the reservoir, the mercury flows through an electromagnetic flow meter and a flow control valve to the apparatus. From there the mercury flows by gravity back to the pump.

The flow meter is a 0·225 in. inside-diameter plastic tube with 0·030 in. diameter stainless-steel wire electrodes mounted perpendicular to the magnetic field. The field is furnished by two permanent bar magnets attached by epoxy cement directly to the tube (see figure 4, plate 2). The flow meter was shielded magnetically from the fringing field of the electromagnet. It was calibrated in place by weighing the mercury pumped through in a given time. The relationship between flow and voltage was linear with a constant of 1·65 in.^{3}/sec/millivolt. We are carrying out a systematic theoretical and experimental investigation of these flow meters.

The magnet was that of the 32·5 in. cyclotron at the University of Chicago. This magnet has 8·75 in. spacing between pole pieces and can obtain a field of 16,000 G which is homogeneous to about 0·1 % in the region of the plates.

3. Procedure

The mercury was cleaned by the process described by Kuzma *et al.* (1964). The plates were cleaned thoroughly before assembly to remove all traces of oil and grease. The apparatus was carefully levelled and a continuous film of mercury established. A photograph of the experiment in the magnet is shown in figure 5 (plate 3). The manometer tubes are clustered on the left and the drain tubes on the right.

The flow was kept on continuously for about an hour before taking measurements in order to establish thermal equilibrium. The experiment consisted of reading the manometers and the flow rate at a series of different magnetic fields and for a number of different spacings between plates.

With narrow spacings $h \lesssim 0{\cdot}015$ in., we found that surface-tension forces gave an irregular pressure profile, making it difficult to establish the zero-flow condition. The 'zero-flow' pressure profile was actually established with a very small flow rate sufficient to break through the free surface formed at the outer edge of the plates. The outermost pressure tap was then taken to be the reference pressure (see the right-hand manometer of figure 6, plate 4).

4. Experimental results

Experimental results are shown in figures 6 to 9. Figure 6 (plate 4) is a photograph of the manometers which were used to measure pressures. This photograph was taken for a constant flow rate of 1·65 in.^{3}/sec and a film thickness of 0·0278 in. The pressures are shown as the magnetic field increases. The magnetic field greatly increases the load-carrying capacity $W = \int p\,dA$, without increasing the pumping power required. At zero field the pressure gradient is scarcely discernible. At higher fields the pressure gradient develops dramatically. At the highest fields magnetic forces dominate viscous forces and a magnetostatic balance $\operatorname{grad} p = \mathbf{j} \times \mathbf{B}$ is obtained. The pressure profile is then logarithmic,

$$p = p_0 + \frac{\sigma B_0^2}{2\pi h} \ln (b/r),$$

where σ is the conductivity, B_0 is the field strength, Q is the volumetric flow rate, b is the radius of the plates, and p_0 is the pressure at $r = b$.

Figure 7 shows the results obtained with a constant flow rate of 0·99 in.^{3}/sec and $h = 0{\cdot}0103$ in. for various magnetic field strengths. They are compared with

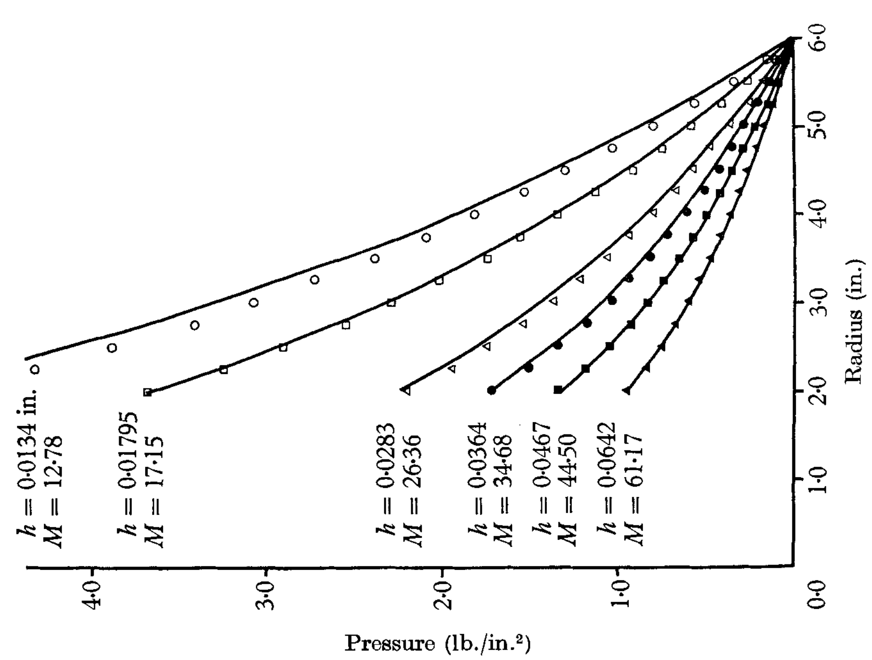

FIGURE 8. Pressure profiles for constant flow rate and constant magnetic-field strength for various film thicknesses. $Q = 1{\cdot}65$ in.^{3}/sec and $B_0 = 14{\cdot}5$ kG. —, theory; ○, experiment.

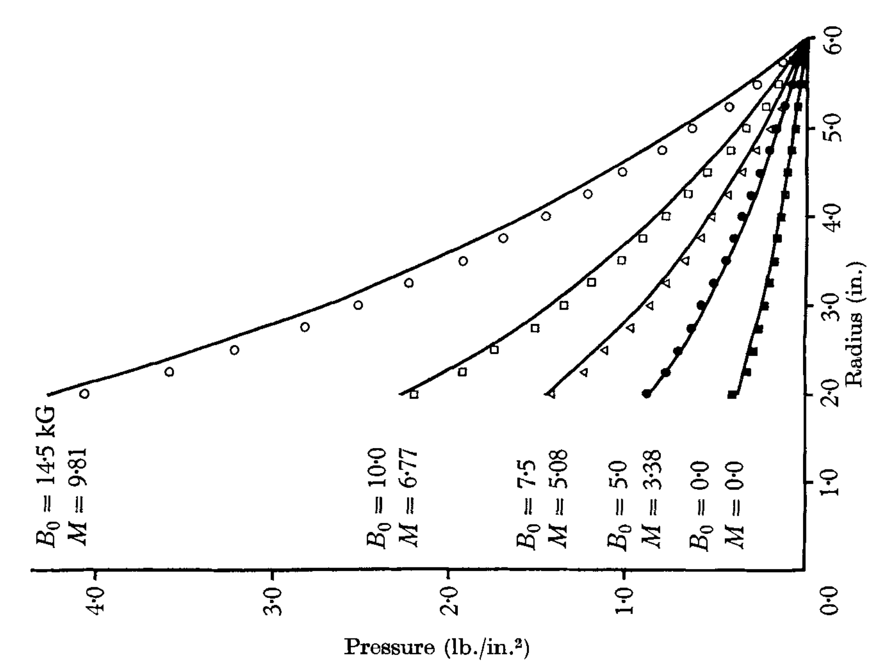

FIGURE 7. Pressure profiles for constant flow rate and constant film thickness for various magnetic-field strengths. $Q = 0{\cdot}99$ in.^{3}/sec and $h = 0{\cdot}0103$ in. —, theory; ○, experiment.

calculations of § 5, equation (11). The trend appears to be accounted for accurately, with some systematic deviations. The origins of these discrepancies are not known. The results, however, are very sensitive to the plate spacing and this could introduce such a systematic error. Figure 8 shows results at constant field and flow rate for various film thicknesses. Figure 9 shows results for constant film thickness and magnetic field and varying flow rates. The pressure in the recess is seen to be higher than that predicted by theory because of entrance effects. In general, theory and experiment appear to be in good agreement.

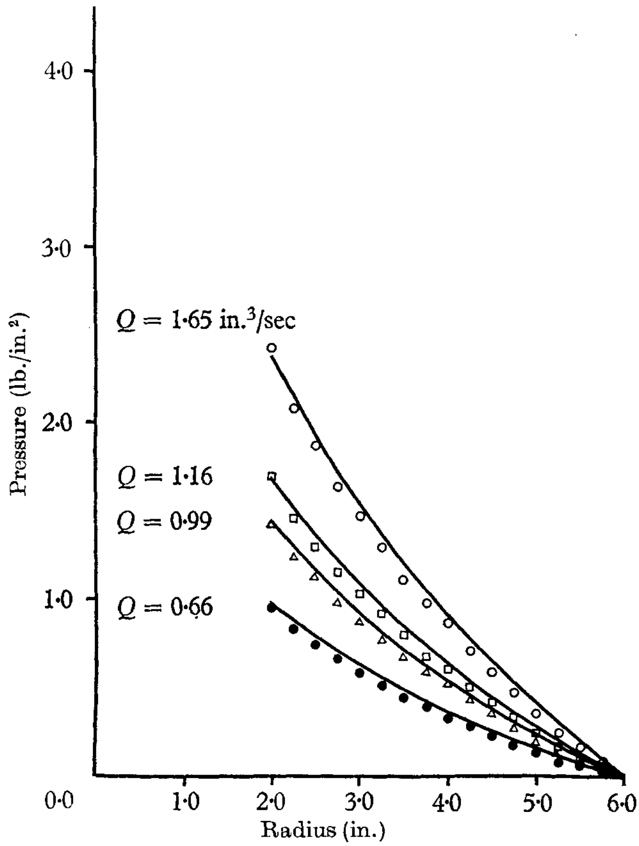

FIGURE 9. Pressure profiles for constant magnetic field and constant film thickness for various flow rates. h = 0·0103 in.; B_0 = 7·5 kG; M = 5·08. —, theory; ○, experiment.

5. Analysis

The analysis for the magnetohydrodynamic lubrication flow between parallel plates in the usual lubrication approximation has been given by Hughes & Elco (1962). In order to interpret the results of the present experiment at the higher flow rates, it is necessary to make allowance for fluid inertia terms. We shall do this by the method used by Kuzma *et al.* (1964).

The equation of conservation of momentum in hydromagnetics is

$$\rho(\mathbf{u}.\nabla\mathbf{u}) = -\nabla p + \mu\nabla^2\mathbf{u} + \sigma(\mathbf{u}\times\mathbf{B})\times\mathbf{B}, \tag{1}$$

since in this case the electric field is zero.

If the usual magnetohydrodynamic lubrication assumptions are made, and the significant inertia term is retained, the momentum equation in the r-direction becomes

$$\rho u\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial r} - \sigma B_0^2 u + \mu\left(\frac{\partial^2 u}{\partial y^2} - \frac{u}{r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right), \tag{2}$$

where u is the radial component of the velocity, y is the vertical co-ordinate and μ is the viscosity.

The velocity profile, u_0, obtained by Hughes & Elco (1962) is used to approximate the terms that are usually neglected. Thus

$$\mu\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u = \frac{\partial p}{\partial r} + \rho u_0\frac{\partial u_0}{\partial r} + \mu\left(\frac{\partial^2 u_0}{\partial r^2} + \frac{1}{r}\frac{\partial u_0}{\partial r} - \frac{u_0}{r^2}\right), \tag{3}$$

where

$$u_0 = \frac{Q}{2\pi r}\frac{M/h}{M - 2\tanh\frac{1}{2}M}\left(1 - \frac{\cosh My/h}{\cosh\frac{1}{2}M}\right). \tag{4}$$

M is the Hartmann number,

$$M = B_0 h\sqrt{(\sigma/\mu)}, \tag{5}$$

and Q is the volumetric flow rate.

If equation (4) is substituted into equation (3), we obtain

$$\frac{\partial^2 u}{\partial y^2} - \left(\frac{M}{h}\right)^2 u = \frac{1}{\mu}\frac{dp}{dr} - \frac{\rho}{\mu}\left(\frac{Q}{2\pi}\right)^2\frac{1}{r^3}\frac{(M/h)^2}{(M - 2\tanh\frac{1}{2}M)^2}\left(1 - \frac{\cosh My/h}{\cosh\frac{1}{2}M}\right)^2. \tag{6}$$

The boundary conditions are

$$u(\pm\tfrac{1}{2}h) = 0.$$

The solution to equation (6) becomes

$$\begin{aligned} u = {} & \frac{1}{\mu}\frac{dp}{dr}\left(\frac{h}{M}\right)^2\left(\frac{\cosh My/h}{\cosh\frac{1}{2}M} - 1\right) \\ & - \frac{1}{r^3}\frac{\rho}{\mu}\left(\frac{Q}{2\pi}\right)^2\frac{(M/h)^2}{(M - 2\tanh\frac{1}{2}M)^2}\left[\frac{h^2}{M^2}\left(\frac{\cosh My/h}{\cosh\frac{1}{2}M} - 1\right)\right. \\ & + \frac{h^2}{2M}\tanh\tfrac{1}{2}M\,\frac{\cosh My/h}{\cosh\frac{1}{2}M} - \frac{hy}{M}\frac{\sinh My/h}{\cosh\frac{1}{2}M} \\ & \left. + \frac{2h^2}{3M^2}\frac{1}{\cosh^2\frac{1}{2}M}\left(\frac{\cosh My/h}{\cosh\frac{1}{2}M} - 1\right) + \frac{h^2}{3M^2}\left(\frac{\cosh^2 My/h}{\cosh\frac{1}{2}M} - \frac{\cosh My/h}{\cosh\frac{1}{2}M}\right)\right]. \end{aligned} \tag{7}$$

From continuity

$$Q = 2\pi r\int_{-\frac{1}{2}h}^{\frac{1}{2}h} u\,dy. \tag{8}$$

The expression for velocity in equation (7) may be substituted into equation (8). After integration, this equation may be arranged to obtain

$$\begin{aligned} \frac{dp}{dr} = {} & \frac{\mu Q}{4\pi}\frac{1}{r}\frac{(M/h)^3}{(\tanh\frac{1}{2}M - \frac{1}{2}M)} + \frac{\rho(M/h)^3}{(2\tanh\frac{1}{2}M - M)^3}\left(\frac{Q}{2\pi}\right)^2\frac{1}{r^3} \\ & \times\left[-\frac{5h}{2} + \frac{5h}{M}\tanh\tfrac{1}{2}M + \frac{3h}{2}\tanh^2\tfrac{1}{2}M - \frac{4h}{3M}\tanh^3\tfrac{1}{2}M\right]. \end{aligned} \tag{9}$$

The boundary condition on the pressure is

$$p(b) = p_0. \tag{10}$$

Equation (9) may be integrated directly to obtain

$$\begin{aligned} p-p_0 = {} & \frac{\mu Q}{4\pi}\frac{(M/h)^3}{\tanh\frac{1}{2}M-\frac{1}{2}M}\ln(r/b) \\ & +\frac{(M/h)^3}{(2\tanh\frac{1}{2}M-M)^3}\frac{\rho Q^2}{8\pi^2}\left(\frac{1}{b^2}-\frac{1}{r^2}\right) \\ & \times\left[-\frac{5h}{2}+\frac{5h}{M}\tanh\tfrac{1}{2}M+\frac{3h}{2}\tanh^2\tfrac{1}{2}M-\frac{4h}{3M}\tanh^3\tfrac{1}{2}M\right]. \end{aligned} \tag{11}$$

If we take the limit of p as M approaches zero, we obtain

$$\lim_{M\to 0} p = \frac{6\mu Q}{h^3}\ln(b/r)-\frac{27\rho Q^2}{140\pi^2h^2}\left(\frac{1}{r^2}-\frac{1}{b^2}\right). \tag{12}$$

This is the same result obtained by Savage (1964) for the flow with no magnetic field.

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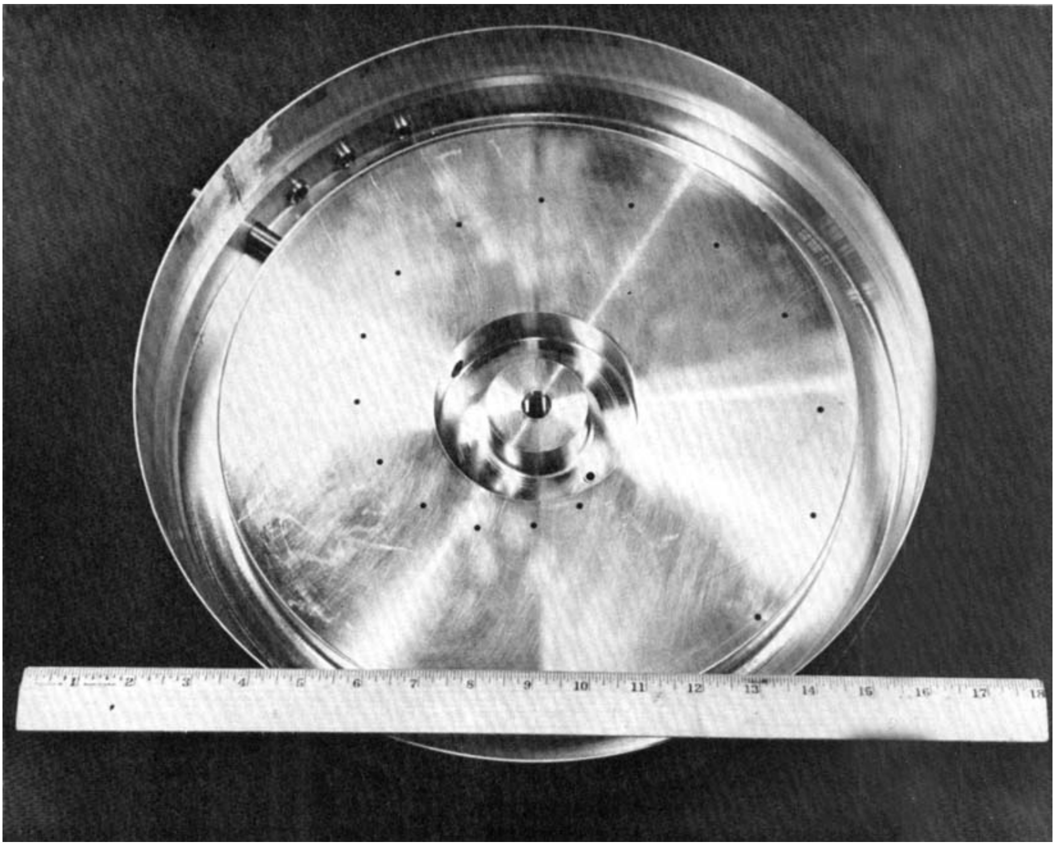

FIGURE 2. Photograph of apparatus.

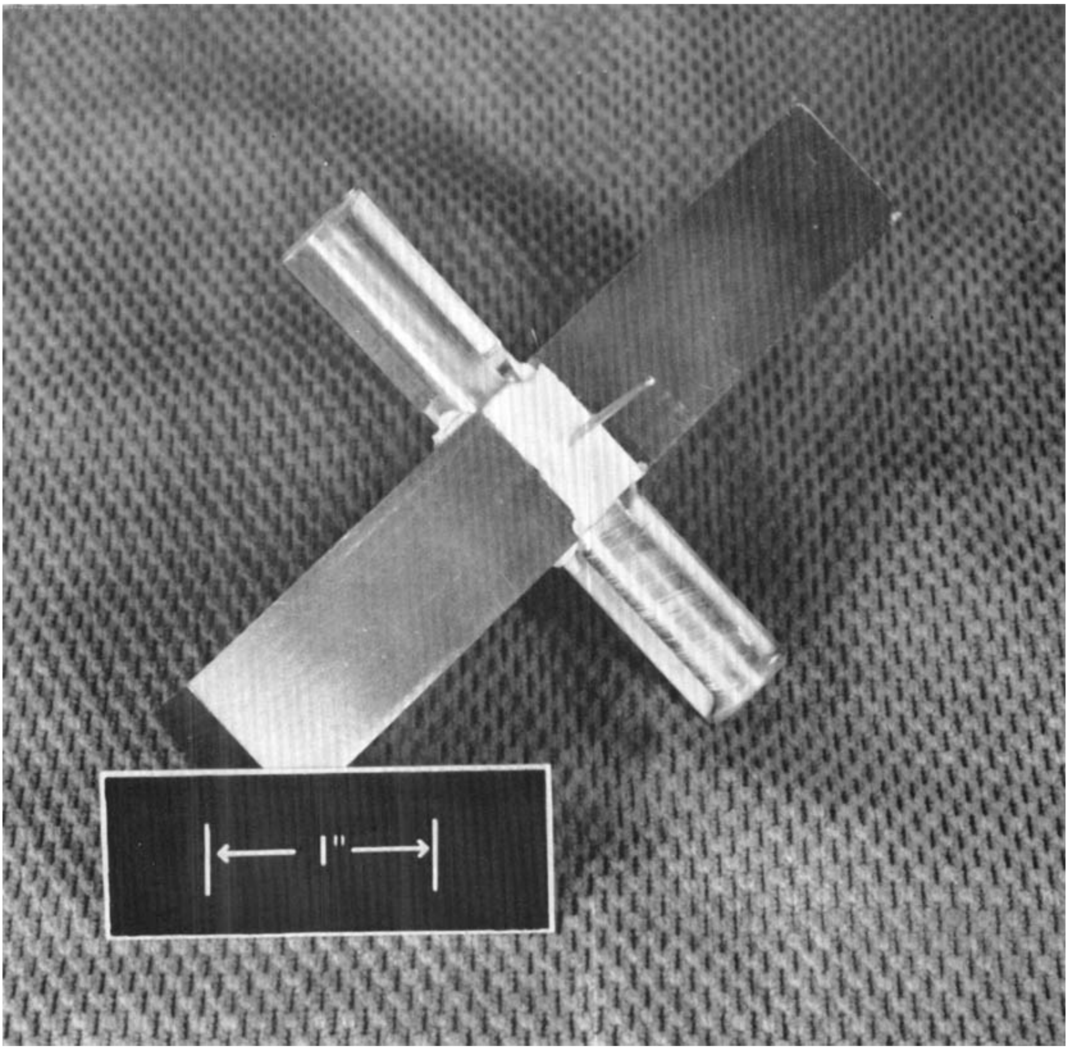

FIGURE 4. Photograph of flow meter.

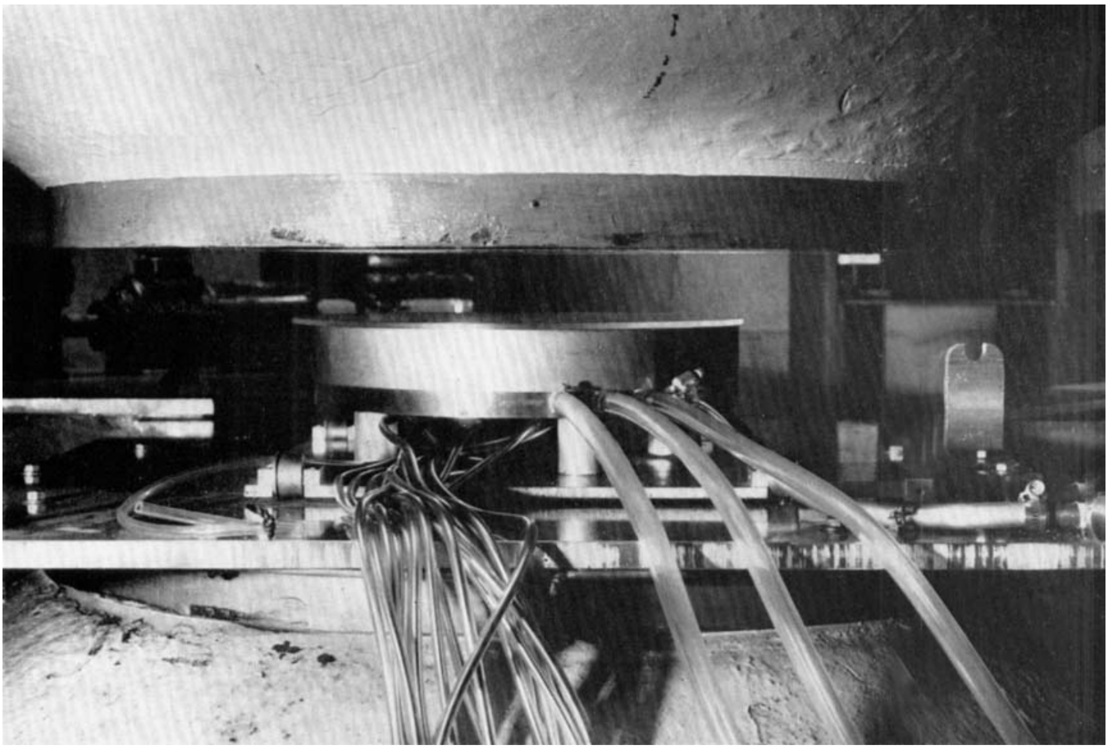

FIGURE 5. Experiment in position between pole pieces.

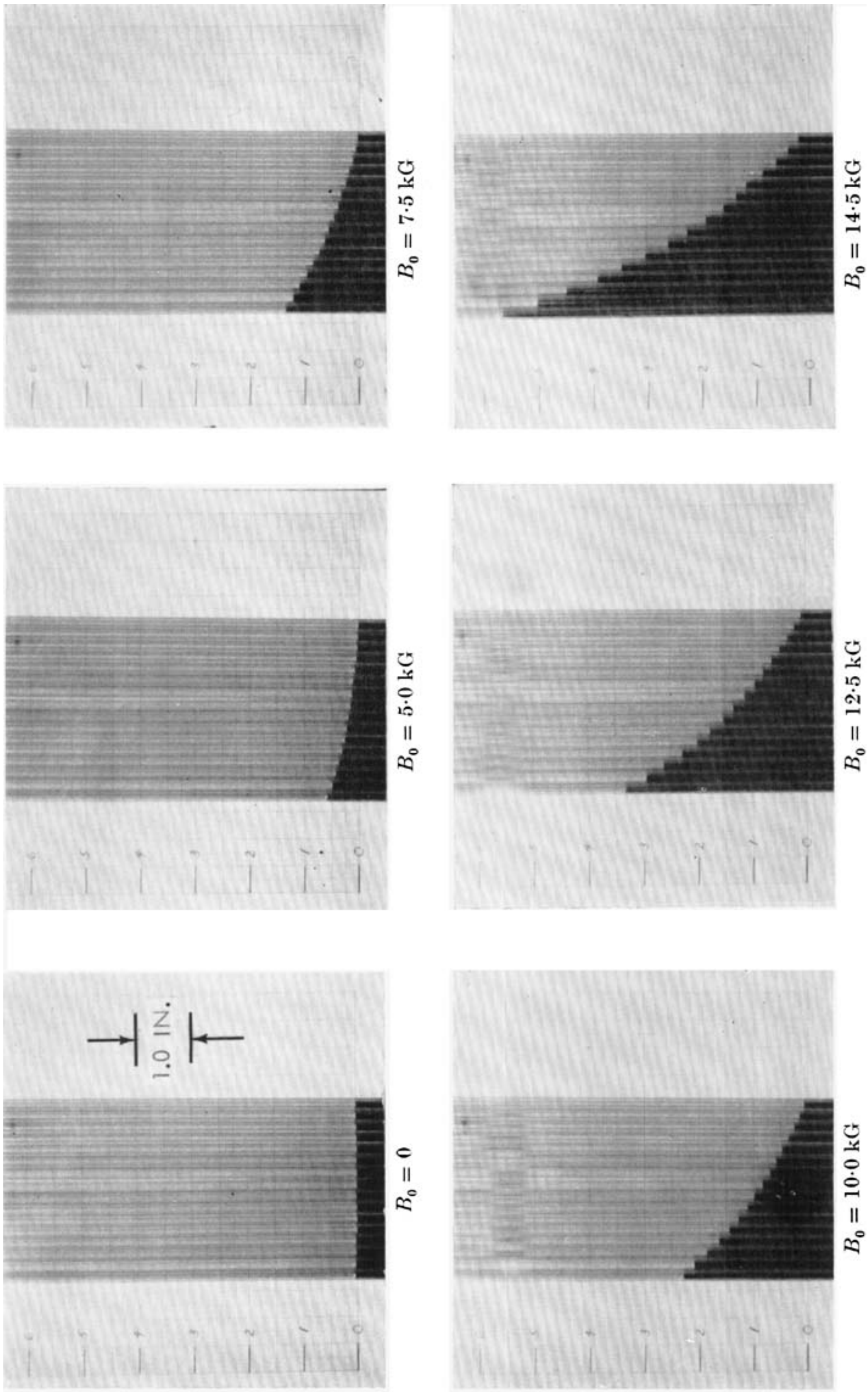

FIGURE 6. Pressure profiles at various magnetic fields for $Q = 1{\cdot}65$ in.^{3}/sec and $h = 0{\cdot}0278$ in.